BEFORE THE
POSTAL RATE COMMISSION

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POSTAL RATE COMMISSION
OFFICE OF THE SECRETARY

POSTAL RATE AND FEE CHANGES, 2000 : DOCKET NO. R2000-1

ANSWER OF UNITED PARCEL SERVICE
WITNESS KEVIN NEELS TO UNITED STATES
POSTAL SERVICE INTERROGATORY
(USPS/UPS-T3-18)
(July 12, 2000)

Pursuant to the Commission's Rules of Practice, United Parcel Service hereby files and serves the answer of UPS witness Kevin Neels to the following interrogatory of the United States Postal Service: USPS/UPS-T3-18.

Respectfully submitted,

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USPS/UPS-T3-18. Please refer to your response to USPS/UPS-T3-3 and your discussion of the TRACS sample design on page 35, lines 16-20 of your testimony. Consider the following hypothetical example of a sample in two strata. Two stop-days are sampled in each stratum with equal probabilities, and there are two subclasses of mail, Subclass X and Subclass Y. The total number of stop-days in each stratum is the same. The hypothetically observed cubic foot miles and mail mix proportions, by subclass, for these four stop-days are shown in the table below.

	Subclass X CFM	Subclass Y CFM	TOTAL CFM	Subclass X Proportion	Subclass Y Proportion
Stratum A					
Stop-Day 1	1500	150	1650	.909	.091
Stop-Day 2	1000	200	1200	.833	.167
Stratum B					
Stop-Day 3	100	10	110	.909	.091
Stop-Day 4	110	50	160	.688	.312
Strata A+B	2710	410	3120	.869	.131

a) Please confirm that the mail mix, as defined in your response to USPS/UPS-T3-3, for the two stop-days sampled in Stratum A, is more uniform than the mail mix for the two stop-days sampled in stratum B. If not confirmed, please explain fully.

b) Using a combined ratio estimator (as is used in TRACS-Highway estimation), please confirm that the proportion of CFM used by Subclass X is 0.869., and for Subclass Y is 0.131. If not confirmed, please explain fully.

c) Assume the cost per test is $100, regardless of strata, and you can afford to take 50 tests. Please determine and show the derivation of the optimum allocation of tests between strata, for the Subclass X proportion of CFM.

d) Assume the cost per test is $100, regardless of strata, and you can afford to take 50 tests. Please determine and show the derivation of the optimum allocation of tests between strata, for the Subclass Y proportion of CFM.

e) Assume the cost per test is $100, regardless of strata, and you can afford to take 50 tests. Please determine and show the derivation of the multivariate allocation of tests between strata, giving equal importance to both subclasses proportion of CFM.

f) Please confirm, for subparts c), d), and e) above, that the optimum allocation results in more tests for Stratum A, the stratum with the more uniform mail mix. If not confirmed, please explain fully.

Response to USPS/UPS-T3-18.

(a) Confirmed.

(b) Confirmed.

(c) The general form of the combined ratio estimator is[1]:

$$\hat{R}_{CX} = \frac{\sum_h N_h \overline{X}_h}{\sum_h N_h \overline{T}_h} \qquad (1)$$

where h is an index of strata, N_h is the number of population units (in this case, stop-days) for stratum h, $\overline{X}_h$ is the average CFM per stop-day for class X in stratum h, and $\overline{T}_h$ is average total CFM per stop-day for stratum h.

1. William G. Cochrane, Sampling Techniques, 3rd Edition, John Wiley & Sons, 1977, p. 165.

In the example there are two strata, each with the same sized population. Under these assumptions, the combined ratio estimator for the class X share of total CFM reduces to:

$$\hat{R}_{CX} = \frac{\overline{X}_A + \overline{X}_B}{\overline{T}_A + \overline{T}_B} \qquad (2)$$

The optimal allocation of sample is that allocation that minimizes the variance of $\hat{R}_{CX}$. To derive this optimum allocation one must first express this variance as a function of the sample sizes. To develop the required variance function i first linearize the equation above by taking a first-order Taylor series expansion. Define the function:

$$g(\overline{X}_A, \overline{X}_B, \overline{T}_A, \overline{T}_B) = \frac{\overline{X}_A + \overline{X}_B}{\overline{T}_A + \overline{T}_B} \qquad (3)$$

Taking a Taylor series expansion then yields:

$$\hat{R}_{CX} \approx g^* + g'_{XA} \cdot (\overline{X}_A - \overline{X}_A^*) + g'_{XB} \cdot (\overline{X}_B - \overline{X}_B^*) + g'_{TA} \cdot (\overline{T}_A - \overline{T}_A^*) + g'_{TB} \cdot (\overline{T}_B - \overline{T}_B^*) \quad (4)$$

where:

$(\overline{X}_A^*, \overline{X}_B^*, \overline{T}_A^*, \overline{T}_B^*)$ is the point around which the expansion is taken,

$$g^* = g(\overline{X}_{A,} \overline{X}_B, \overline{T}_A, \overline{T}_B)$$

$$g'_{XA} = \frac{\partial g}{\partial \overline{X}_A} = \frac{1}{\overline{T}_A + \overline{T}_B}$$

$$g'_{XB} = \frac{\partial g}{\partial \overline{X}_B} = \frac{1}{\overline{T}_A + \overline{T}_B}$$

$$g'_{TA} = \frac{\partial g}{\partial \overline{T}_A} = -\frac{\overline{X}_A + \overline{X}_B}{(\overline{T}_A + \overline{T}_B)^2}$$

$$g'_{TB} = \frac{\partial g}{\partial \overline{T}_B} = -\frac{\overline{X}_A + \overline{X}_B}{(\overline{T}_A + \overline{T}_B)^2}$$

Rearranging equation (4) yields:

$$\hat{R}_{CX} \approx C + g'_{XA}\overline{X}_A + g'_{XB}\overline{X}_B + g'_{TA}\overline{T}_A + g'_{TB}\overline{T}_B \qquad (5)$$

where C is a constant containing no random variables.

Application of the standard formula for the variance of a linear combination of random variables yields:

$$Var(\hat{R}_{CX}) \approx (g'_{XA})^2 Var(\overline{X}_A) + (g'_{XB})^2 Var(\overline{X}_B) + (g'_{TA})^2 Var(\overline{T}_A) + (g'_{TB})^2 Var(\overline{T}_B)$$

$$+ 2g'_{XA}g'_{XB}Cov(\overline{X}_A, \overline{X}_B) + 2g'_{XA}g'_{TA}Cov(\overline{X}_A, \overline{T}_A) + 2g'_{XA}g'_{TB}Cov(\overline{X}_A, \overline{T}_B) \qquad (6)$$

$$+ 2g'_{XB}g'_{TA}Cov(\overline{X}_B, \overline{T}_A) + 2g'_{XB}g'_{TB}Cov(\overline{X}_B, \overline{T}_B) + 2g'_{TA}g'_{TB}Cov(\overline{T}_A, \overline{T}_B)$$

Independence of the samples in the two strata implies that:

$$Cov(\overline{X}_A, \overline{X}_B) = Cov(\overline{X}_{A,}\overline{T}_B) = Cov(\overline{T}_A, \overline{T}_B) = 0 \qquad (7)$$

The sample mean variances and covariances are given by:

$$Var(\overline{X}_A) = S_{X_A^2} \cdot \frac{N - n_A}{Nn_A}$$

$$Var(\overline{T}_A) = S_{T_A^2} \cdot \frac{N - n_A}{Nn_A} \qquad (8)$$

$$Cov(\overline{X}_A, \overline{T}_A) = S_{XT_A} \cdot \frac{N - n_A}{Nn_A}$$

where N is the population size (the same for both strata) and $S_{X_A^2}, S_{T_A^2}$ and S_{XT_A} are the population variances and covariance. The formulas for stratum B, of course, are analogous.

The question specifies that the sample size for the two strata must sum to 50. Thus:

$n_B = 50 - n_A$ (9)

Substituting (7), (8) and (9) into (6) yields:

$$Var(\hat{R}_{CX}) \approx ((g'_{XA})^2 S_{X_A^2} + (g'_{TA})^2 S_{T_A^2} + 2g'_{XA}g'_{TA}S_{XT_A})\frac{N-n_A}{Nn_A}$$

$$+((g'_{XB})^2 S_{X_B^2} + (g'_{TB})^2 S_{T_B^2} + 2g'_{XB}g'_{TB}S_{XT_B})\frac{N-(50-n_A)}{N(50-n_A)} \quad (10)$$

or,

$$Var(\hat{R}_{CX}) \approx A_1\frac{N-n_A}{Nn_A} + A_2\frac{N-(50-n_A)}{N(50-n_A)} \quad (11)$$

where:

$$A_1 = (g'_{XA})^2 S_{X_A^2} + (g'_{TA})^2 S_{T_A^2} + 2g'_{XA}g'_{TA}S_{XT_A} \quad (12)$$

$$A_2 = (g'_{XB})^2 S_{X_B^2} + (g'_{TB})^2 S_{T_B^2} + 2g'_{XB}g'_{TB}S_{XT_B} \quad (13)$$

Dropping the finite population correction simplifies (11) further to:

$$Var(\hat{R}_{CX}) \approx \frac{A_1}{n_A} + \frac{A_2}{50-n_A} \quad (14)$$

To determine the optimal allocation of sample I then calculate:

$$\frac{\partial Var(\hat{R}_{CX})}{\partial n_A} = \frac{-A_1}{n_A^2} + \frac{A_2}{(50-n_A)^2} = 0 \quad (15)$$

Rearranging (15) yields a quadratic expression for n_A:

$$(A_1 - A_2)n_A^2 - 100A_1n_A + 2500A_1 = 0 \quad (16)$$

From the information presented in the table I calculate the following quantities:

$\overline{X}_A = 1250$

$\overline{T}_A = 1425$

$\overline{X}_B = 105$

$\overline{T}_B = 135$

$S_{X_A^2} = 125000$

$S_{T_A^2} = 101250$

$S_{X_B^2} = 50$

$S_{T_B^2} = 1250$

$S_{XT_A} = 112500$

$S_{XT_B} = 250$

Inserting these quantities into equation (16) I calculate that the optimal allocation assigns 38 of the 50 samples to stratum A.

(d) Since $\hat{R}_{CY} = 1 - \hat{R}_{CX}$, $Var(\hat{R}_{CY}) = Var(\hat{R}_{CX})$. Hence, the allocation that minimizes one variance will also minimize the other.

(e) Since the sample allocation derived in part c) minimizes the variance of both ratio estimates, that same allocation would be optimal here.

(f) Confirmed.

DECLARATION

I, Kevin Neels, hereby declare under penalty of perjury that the foregoing answers are true and correct to the best of my knowledge, information, and belief.

Kevin Neels

Kevin Neels

Dated: 7/12/00

CERTIFICATE OF SERVICE

I hereby certify that I have this date served the foregoing document by first class mail, postage prepaid, in accordance with Section 12 of the Commission's Rules of Practice.

John E. McKeever

John E. McKeever
Attorney for United Parcel Service

Dated: July 12, 2000
Philadelphia, Pa.

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